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AN EARLY REFERENCE TO PERFECT NUMBERS? SOME NOTES ON EUPHORION, *SH* 417[1]

[Elias] *In Porphyrii Isagogen* 8.21 (Paris. gr. 1939 f. 26v, Monac. gr. 399 f. 165 r): *τούτοις δὲ τοῖς τελείοις ἀριθμοῖς μαρτυρεῖ καὶ ὁ Εὐφορίων ἐν τῇ Μοψοπίᾳ λέγων οὕτω·*

σφοῖσιν ἶσοι μελέεσσι, τὸ καὶ καλέουσι τελείους

τούτοις δὲ τοῖς τελείοις ἀριθμοῖς Par. *ante corr.*: *τούτοις δὲ τοὺς τελείους ἀριθμοὺς* Monac., Par. *post corr.* *μαρτιρεῖ* codd., corr. Westerink *Εὐφωρίων* codd.: corr. Westerink *ἐν τημοναψοποιΐα* codd., corr. Westerink *σφοῖσιν ἶσοι μελέεσσι* (vel *ἶσοι σφοῖσι μέλεσσι*) Westerink: *σφοῖσι μελέσσιν ἶσοι* E. Hussey: *σφίσιν οἷσι μελέσσι* codd. *τὸ καὶ καλέουσι τελείους* (vel *τὸ καί τέλεοι καλέονται*) Westerink: *τῷ καὶ τέλεοι καλέονται* E. Hussey: *τῷ καὶ καλέονται τέλειοι* codd.

These perfect numbers are also attested by Euphorion in the *Mopsopia*, when he says: ' . . . equal to his (their) limbs, with the result that they are called perfect'.

Euphorion *SH* 417 (fr. 36 Van Groningen) deserves to be better known.[2] A curiosity in itself—an apparent poetic reference to number theory—it is also, potentially, one of our earliest references to Euclidean material. On the authority of a late commentator on Aristotle, Euphorion, a mid-third-century B.C. Euboean poet who was also active in Athens and Antioch, is said to have mentioned perfect numbers—i.e. numbers which equal the total of all their factors, including 1 (but obviously excluding the number itself). It is a pity that the context in Euphorion does not survive, and that the line is only preserved, and indeed interpreted, by so late a source. But the wording of the fragment—if Westerink's restoration of its various corruptions (again, a pity) is plausible—would strongly suggest a reference to the notion of perfect number. The fragment has been known since Westerink published it in 1960, and was included both in Van Groningen's edition of Euphorion in 1977 and in *Supplementum Hellenisticum* (1983). But its implications have still not been discussed, and when David Fowler came to gather the evidence for references to Euclidean material in and after the third century B.C. in *The Mathematics of Plato's Academy*, his attention, unsurprisingly, was not drawn to it.[3] Euphorion has had a bad press, as a poet of rebarbatively obscure myth and intractable vocabulary; yet he holds some interest, and we may be missing more insights into the intellectual life of the Hellenistic period which the perverse intelligence and baneful wit of the fragments display.

[1] I should like to thank the three people who helped in the construction and criticism of this paper: David Fowler, Edward Hussey, and the anonymous referee of *CQ*.

[2] The fragment was not known to Powell. However I cite the other poetic fragments from Powell's *Collectanea Alexandrina* and testimonia for Euphorion's life from Van Groningen.

[3] D. H. Fowler, *The Mathematics of Plato's Academy: A New Reconstruction* (Oxford, 1987). The earliest reference to Euclid which he finds is presented on p. 208, and consists of ostraka from Elephantine in Upper Egypt, which deal with the results of *El.* 13.10 and 16 (Pack[2] 2323). Like our material, they come from the third quarter of the third century B.C. They do not follow our text of the *Elements*, but seem to constitute an attempt to work it through to the writer's own personal satisfaction.

SOURCE

The fragment was discovered by L. G. Westerink and published as 'A new fragment of Euphorio', *Mnemosyne* s.4. 13 (1960), 329–30. Van Groningen (fr. 36) misleadingly implied that it derived from the Aristotle commentator, David, from a fuller text of his discussion of perfect numbers.[4] But it does not come from David; rather, it is from a text that goes under the name of pseudo-Elias (henceforth [Elias]), published by Westerink as *Pseudo-Elias (Pseudo-David): Lectures on Porphyry's Isagoge* (Amsterdam, 1967). This is an obscure and undistinguished text, curious in the density of its medical learning. As Westerink's introduction says (pp. xi–xvi), it overlaps with material from other Aristotle commentators, especially Elias and David, to both of whom it is attributed by titles in different manuscripts. Probably the author is identical with neither, though does belong to the same teaching tradition. Westerink gives his *termini* as the end of the sixth century, on the one hand, and the closure of the university of Constantinople in 726, on the other.

The beginning of the text is lost. What we have[5] begins with a recapitulation of the conventional theme of the reason for the six ὁρισμοί (definitions) of philosophy, and proceeds with a derivative account of even and odd numbers and their various species, even-times-even, odd–even, and even–odd. This jejune account is quite unoriginal, deriving mostly from David and Nicomachus. [Elias] then leads into his discussion of perfect and imperfect numbers (8.7); he compares perfect numbers with perfectly formed humans; ὑπερτελεῖς or over-perfect numbers (whose factors add up to more than the number itself) with 'over-endowed' monsters like the hundred-handed Briareus; and ὑποτελεῖς or less-than-perfect numbers (whose factors add up to less) with the deficient, like monophthalmic Cyclops. He then defines and exemplifies these three types of number (8.12–15); remarks on the rarity of perfection in numerology as in everything else, for only one such number is to be found among the tens, the hundreds, the thousands, and the ten-thousands; and observes that perfect numbers end in 6 and 8 alternately (8.16–20).[6] Elias now relaxes the turgidly elementary maths: enter Euphorion (8.21). These perfect numbers, [Elias] says (not indicating *which*), are alluded to in Euphorion's *Mopsopia*. The lecture ends with a restatement: how reasonable it is, then, and how natural, that there should be six definitions of philosophy.

Unfortunately the line is corrupt. The title of the poem, the *Mopsopia*—independently attested (frr. 34–6 P.)—is the least of the difficulties; as Westerink says, -μονα- in the manuscript could easily have arisen from the word μονάς, frequent in the context in [Elias]. More serious is that ἴσοι is Westerink's restoration. But it seems fairly secure. The first half of the line must explain *why* some numbers are called 'perfect'; ἴσος supplies the required sense, and it hardly matters that it is not the word [Elias] himself uses.[7] If it is right, Euphorion could be echoing the standard definition

[4] See A. Busse, *Davidis Prolegomena et in Porphyrii Isagogen Commentarium* (Berlin, 1904) = *CAG* XVIII.2. The discussion of perfect numbers is at pp. 22.18–35 Busse.

[5] The beginning of the text, including the section on perfect numbers, is represented only by the manuscripts Paris gr. 1939 and Monac. gr. 399; on the manuscripts see Westerink, *Pseudo-Elias*, pp. vii–viii.

[6] This is true of the first five perfect numbers—Nicomachus knew the first four—but it ceases to be true afterwards: see T. Heath, *History of Greek Mathematics* (henceforward *HGM*) (Oxford, 1921; repr. New York, 1981), vol. I, p. 74.

[7] [Elias] has been using different verbs, τὰ μέρη συντιθέμενα ἀποτελοῦσι τὸ ὅλον, and the like.

of perfect numbers given in Euclid, *El.* 7 *Def.* 23: τέλειος δὲ ἀριθμός ἐστιν ὁ τοῖς ἑαυτοῦ μέρεσιν ἴσος ὤν.[8]

Nearly all [Elias] says about perfect numbers comes from Nicomachus. The latter flourished around A.D. 100; a popularizer, he was more interested in the 'mystic rather than the mathematical side of the theory of numbers'.[9] Though 'not really a mathematician', he made mathematics as easy and accessible as his overheated and mystagogic style would allow, and [Elias] names him elsewhere as one pre-eminent in ἀριθμητική (19.30). He, like [Elias], exemplifies over-perfect numbers by Briareus, and quotes the Hesiodic hexameter about the Cyclops to illustrate the less-than-perfect variety.[10] When [Elias] compares the pattern of alternate endings in 6 and 8 to that of a sieve, the word would seem to have been suggested by the Eratosthenic κόσκινος which Nicomachus has described in the immediately preceding section (1.13). The one thing Nicomachus does not mention is Euphorion's line. But might an earlier, or fuller, text or commentary have done so? It is also interesting that the quotation from Hesiod found in this section seems to be the only occasion in the *Introductio Arithmetica* when Nicomachus turns to poetry. Was there a tradition of exemplifying perfect numbers from poetry, a tradition in which the lines of both Hesiod and Euphorion were originally quoted? Or is the quotation from Euphorion [Elias]'s own independent addition? Whatever his source, he does not look very sure about what use he wants to make of the quotation, which is oddly tacked on at the end, reminiscent of an afterthought, or as if the author was so pleased with a sudden insight into an obscure hexameter that he could not resist writing it down. [Elias] cites numerous other poets, usually passages which have entered the mainstream of Aristotle commentary.[11] But a few citations cannot be paralleled from the other commentators, citations which might have come from lexica, for example, or anthologies.[12] Does Euphorion's line come from a lost commentary or lexicon entry? We do not know. Edward Hussey, observing the density of the medical learning in [Elias], suggests to me by letter that the author perhaps got the line of Euphorion from some medical compilation; indeed, the number six was thought by some to have medical implications.[13] But for the time being at least, the source of the line is a mystery.

EUPHORION'S KNOWLEDGE OF PERFECT NUMBERS

It is far from clear that Euphorion is referring to a text of Euclid himself, rather than that he is paraphrasing a mathematical cliché worded in this way before Euclid laid it down. If one more competent in Greek mathematics than I were to assert that perfect numbers could not have been *defined*, *worded*, in this way before Euclid, then we would have a reference to the—or *a*—text of Euclid himself from the middle of the third century B.C. If, on the other hand, perfect numbers were conceptualized and defined in this way before Euclid, then Euphorion might equally well be referring to a notion he had picked up elsewhere. This is the central question. Had Euphorion

8 *Euclidis Elementa*, ed. I. L. Heiberg, rev. E. S. Stamatis (Leipzig, 1970), vol. II, p. 105.

9 Heath, *HGM*, vol. I, pp. 98–9.

10 David 8.10, Nicomachus 1.15.1.

11 Callimachus' Cleombrotus epigram (*AP* 7.471 = *Ep.* 23 Pf. = *HE* 1273–6) in [Elias] 12.5; David, p. 31.30–3 Busse. Theognis 175–6 in [Elias] 13.4; David, p. 32.21–2; Elias, p. 15.17–18 Busse in *CAG* XVIII.1. There are also tags from the tragedians and Homer.

12 ([Elias] 19.23) cites Ap. Rhod. 3.1323, also in *Et.Mag.* 43.47, s.v. ἄκαινα.

13 E.g. [Iamblichus] *Theologoumena Arithmeticae*, ed. V. de Falco (Leipzig, 1975), listing πανάκεια (p. 50.2–3) and ὑγίεια (p. 48.21) among the names for six.

read Euclid; or have we here evidence that the idea may have been in more general circulation?

The facts, such as they are, are well-known. τέλειος ἀριθμός first appears in Euclid *El.* 7 *Def.* 23: τέλειος ἀριθμός ἐστιν ὁ τοῖς ἑαυτοῦ μέρεσιν ἴσος ὤν. At 9.36 Euclid shows the law of their formation.[14] Perfect numbers are next mentioned by Theon of Smyrna pp. 45.9ff. and Nicomachus 1.16, Iamblichus *In Nicom.* 32.20ff., who add information about hyper- and hypoperfect numbers.[15] It is presumably from Nicomachus that the doctrine becomes a topos in the Aristotle commentators.

The date of Euclid is irremediably obscure; the consensus tends to be that his *floruit* was around 300 B.C., and he is associated with a school in Alexandria.[16] Euphorion is not known to have had any direct connection with Alexandria,[17] though he certainly knew Alexandrian poetry. It does not seem at all unlikely that he *might* have known the work of a recent Alexandrian mathematician; but there is another place where he might have picked up the doctrine of perfect numbers, and that is Athens. He almost certainly received training there in philosophy in the middle of the third century B.C., and it is likely that Euclid was educated in Athens too.[18] It is surely at least as likely that Euphorion got his perfect numbers from his education in Athens as that he got them straight from Euclid. A further argument might be derived from the single-word fragment 151 P., cited without context. It contains the single-word ἐλλιπέες. It might mean simply 'defective', but the word is extraordinarily rare in poetry, and I can parallel it only by ἐλλιπές in Euangelus, a comic poet of perhaps the third century B.C.[19] The very rarity of the general meaning 'defective' could, as Westerink saw, be an argument for assigning it a technical sense in Euphorion, referring to numbers which are 'less than perfect', those whose factors add up to less than the number itself. But this notion is absent from the text of Euclid himself. It is first found in Theon of Smyrna and Nicomachus, in their discussions of perfect numbers. Here would be evidence that Euphorion did not know the contents of Euclid as we know them, but had picked up a notion which had gained general mathematical currency in the academies of his day.

PRE-EUCLIDEAN PERFECT NUMBERS?

The Pythagorean perfect triangle, the τετρακτύς, was a pebble-figure consisting of ten individual pebbles arranged in rows of $1 + 2 + 3 + 4$.[20] Later commentators point

[14] When the sum of the sequence of duplications starting from one (1, 2, 4, and so on) is a prime number, then that number, multiplied by the last term of the series, will result in a perfect number. The first two perfect numbers are produced thus: $6 = (1 + 2) \times 2$, and $28 = (1 + 2 + 4) \times 4$. On the other hand $1 + 2 + 4 + 8$ results in 15, which is not a prime number, so that $15 \times 8 = 120$, which is not perfect.

[15] See Heath, *HGM*, vol. I, pp. 74–5 and his commentary on Euclid *El.* 7 *Def.* 23 and 9.36.

[16] See on his *Life and Works* in *Dictionary of Scientific Biography*, ed. C. C. Gillespie (New York, 1970–8), vol. II, pp. 414–37 (I. Bulmer-Thomas), esp. pp. 414–15, and on the transmission of the *Elements*, ibid., pp. 437–59 (J. Murdoch), esp. pp. 437–8. Our main sources are Proclus, *In Primum Euclidis*, p. 68.6–23 Friedlein, and Pappus *Collection* VII.35, II, p. 678.10–12 Hultsch.

[17] *Pace* P. Treves, *Euforione e la storia ellenistica* (Milan, 1955), pp. 48–54; see the review by P. M. Fraser, *Gnomon* 28 (1956), 580–1.

[18] See Euphorion's *Suda* entry (ε 3801) and (a reference to Athenian citizenship) Helladius ap. Photius cod. 279 p. 532 b18 B = Testt. 1 and 3 Van Groningen; for Euclid, see Bulmer-Thomas, art. cit. (n. 16).

[19] *An.* 1.3 (*PCG* V 184–5 Euangelus).

[20] For instance, see *Carm. Aur.* 47; Sext. Emp. *Adv. Math.* 7.94; Hierocles on *Carm. Aur. FPG* I p. 465 Mullach.

out the difference between the various applications of the term 'perfect number' in antiquity.[21] The Pythagoreans do not seem to have assigned any particular importance to the number 6, though, as Burkert observes, a riddle in an epigram tells us that they did list 28 pupils of Pythagoras, and there were also supposed to have been 28 members of the Academy.[22] The epigram presents the number of the followers of Pythagoras precisely in the form of the total of the factors of 28, yet it is clearly late and need not represent early Pythagorean teaching. It belongs to a group (*AP* 14.1–4) in which a larger number is represented as a total of smaller numbers; the others are not perfect, but the method is the same, additive, one as the way in which perfect numbers are produced. It is intriguing, but can hardly be used as evidence for the state of number theory in or before the third and fourth centuries B.C.

Could the Pythagoreans be responsible for the notion of perfect number which led to the designation of 6, 28 and the rest as τέλειος? It is *a priori* very likely that it was around before Euclid. By letter, David Fowler comments that 'the suggestion that . . . perfect numbers were *not* known much before Euclid . . . would be seen as startlingly revolutionary by most, perhaps all people working on Greek mathematics, though . . . the evidential basis for it is very tenuous'. One way in which the notion could have been discovered is proposed by Taisbak.[23] This hypothesis depends on addition and duplication, and presupposes the use of Egyptian-style multiplication tables by the Greeks, but, like Euclid's rule for finding them, it depends essentially on the fact that a perfect number is produced by multiplying the total number achieved by adding together the terms in the series of duplications 1, 2, 4, 8, and so on, *whenever this total number is itself prime*, by the last term in the series. Taisbak concludes that a Pythagorean *could* be a candidate for discovery of this idea, and Becker had already laid the notion at the same door. In an article that is often held to have settled the matter once and for all, he argued that Euclid 7.21–34 and definitions 7.6–11, which concern odd and even numbers, are a group of early Pythagorean propositions preserved out of reverence for their antiquity, despite their lack of connection with anything else in books 7–9 (and their lack of profundity in comparison with the rest of those books), and whose purpose was originally to lead up to the proposition on perfect numbers.[24] Of the method of producing them, the dyadic process of continuous doubling, he says that 'the Egyptian method of calculation should certainly be assumed as its origin'.

Burkert, however, is sceptical. Plato's Academy is the better place to look; Becker's attribution of the notion to the Pythagoreans contains the seeds of its own destruction, because it presupposes the use of algebraic notation which would have been unavailable to those who proceeded by such empirical means as black-and-white counters; the concepts conveyed by the algebraic notation simply could not have been developed by those to whom such notation was unknown.[25] Burkert, however, may have been unduly impressed by the more technical sections of Becker's article. It is indeed hard to conceive that the Pythagoreans would have had the means to express

21 Theon of Smyrna, p. 46.12–13, Nicom. ap. Iamblich. *Theol. Arithm.*, p. 44.15.

22 W. Burkert, *Lore and Science in Ancient Pythagoreanism* (Cambridge, MA, 1972), p. 431, n. 28; *AP* 14.1; Athen. 1.4e.

23 C. M. Taisbak, 'Perfect Numbers—A Mathematical Pun?', *Centaurus* 20 (1976), 269–75.

24 O. Becker, 'The Theory of Odd and Even in the Ninth Book of Euclid's *Elements*, An Attempt to Reconstruct its Original Form', trans. in *The Graduate Philosophy Journal* 16. 1 (1992), 87–110, originally published as 'Die Lehre vom Geraden und Ungeraden im neunten Buch der euklidischen Elemente', *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* Abteilung B3 (1934), 533–53.

25 Burkert (n. 22 above), p. 435.

some of the ideas in Becker's article (take the formulations on p. 95). Neither, for that matter, would the Platonic Academy. But neither would they need to. Becker deliberately and explicitly constructed his proofs in such a way that they could have been carried out with counters.[26]

The real problem is the identity of the Pythagoreans, even the possibility of mapping out their intellectual interests to the extent of being able to assert or to deny that they were the ones who first conceptualized perfect numbers. But the notion is surely not all that difficult, and, at least if perfect numbers are calculated by Euclid's method, then it must follow the notion of prime numbers. The term *πρῶτος ἀριθμός* seems first to occur in an extant text in Aristotle.[27] The notion might go back at least to the early Pythagorean Philolaus, who flourished in the second half of the fifth century, but the testimonium which is cited as evidence for this is far from unambiguous.[28] Later, but more securely, Archytas' music-theoretic proposition that 'no mean number may be interpolated in an epimoric interval' depends upon the idea of numbers which are prime to each other.[29] It does not seem difficult to assign the discovery of perfect numbers to Pythagoreans, at least from the time of Archytas (*c.* 400 B.C.), nor to hold that, if the concept *was* current in the Academy of Euclid's and Euphorion's generations, it owed its existence there to the thought of Pythagoras' later followers.

THE *MOPSOPIA*: THE PERFECT DISMEMBERMENT?

'. . . Equal to his (or their) *μέλη*, with the result that they are called "perfect".'[30] Do we know anything at all about the context?

The only clue is the title of the work, *Mopsopia*. Euphorion's *Suda* biography tells us that this was the title of one of the poet's *βιβλία ἐπικά* (so, if it was the name of a whole poetry book, it could have contained poems with their own individual titles); it adds that it dealt with Attica. Fr. 34 P. runs through some of Attica's earlier names. Another fragment seems to have dealt with the Orphic legend of the death of Dionysus at the hands of the Titans: fr 36 P., from Philodemus, is evidence for some treatment of the story that Dionysus, in one of his incarnations, was torn apart by Titans and reassembled by Rhea.[31] This is perhaps to be brought into relation with fr. 13 P., cited from the *Dionysus*, apparently on the roasting of the dismembered god and his burial in Delphi.[32] That the *Dionysus* was perhaps part of the *Mopsopia* was

[26] On Pythagorean 'pebble arithmetic', see C. A. Huffman, *Philolaus of Croton, Pythagorean and Presocratic* (Cambridge, 1993), pp. 185–6.

[27] Arist. *P.A.* I 4, p. 73a 40–b1 and II 13, p. 96a 35–7; *Top.* *Θ* 2, p. 157a 39; *Met.* A 6, 987b 33.

[28] Heath, *HGM*, vol. I, p. 72; O. Becker, *Das mathematische Denken der Antike* (Göttingen, 1957), pp. 46–7 asserts that Philolaus mentioned prime numbers. The reference to Philolaus is in Speusippus ap. *Theol. Arithm.* 82.10 (= *Vors.* I^3, p. 304.5, but not included in the sixth edition), but it is far from clear that it is truly well-informed; see Huffman (n. 26 above), pp. 359–63. Thymaridas is also supposed to have made use of the notion, Iambl. *In Nicom.*, p. 27.4.

[29] See B. L. Van der Waerden, *Science Awakening* (Groningen, 1954), vol. I, p. 111. He argues that the contents of the whole of Book VII of the *Elements* should be attributed to the Pythagoreans before Archytas.

[30] *σφός* could, of course, refer to the third-person singular: *KB* I.602 and Schwyzer II.204–5.

[31] Philodemus *De Piet.* 44, p. 16.1 Gomp. = *Orph.* fr. 36 Kern: *[πρώτην τού]των τὴν ἐκ μ[ητρός], ἑτέραν δὲ τ[ὴν ἐκ] τοῦ μηροῦ, [τρί]την δὲ τὴ[ν ὅτε δι]ασπασθεὶς ὑπὸ τῶν Τιτάνων Ῥέ[ας τὰ] μέλη συνθε[ίσης] ἀνεβίω[ι]. καὶ [ἐν] Μοψοπίαι δ' Εὐ[φορί]ω[ν ὁ]μολογεῖ [τού]τοις, [ὁ] δ' Ὀρ[φεὺς ἐν Ἅιδου] καὶ πάντα [χρόνον] ἐνδιατρε[ίβειν]* (suppl. Gomperz et Wilamowitz, *Hermes* 33 [1898], 521).

[32] *ἐτιμᾶτο δὲ καὶ Διόνυσος ἐν Δελφοῖς σὺν Ἀπόλλωνι οὑτωσί· οἱ Τιτᾶνες τὰ Διονύσου*

suggested in dissertations by Schultze and Scheidweiler and has been repeated with approval since then.[33]

σφοῖσιν ἴσοι μελέεσσι seems to recall Euclid's τοῖς ἑαυτοῦ μέρεσιν ἴσος, punning on μέλος and μέρος. μέλη could be 'songs' or 'limbs', but the latter is much easier to understand with ἴσος, and μέλος and μέρος are also linked in the sense of body-parts in anatomical contexts.[34] We saw, too, that Nicomachus and, following him, [Elias], exemplified hyperperfect and defective numbers by analogies with bodies which had too many or too few parts; so it seems possible that Euphorion himself used some sort of analogy between perfect numbers and body parts. In view of the poem's known connection with Dionysus, Barigazzi suggested that our fragment might have to do with his dismemberment at the hands of the Titans.[35] The translation would then be something like, '(pieces) equal to (the number of) his limbs'—i.e. he was divided limb from limb, piecemeal, μελεϊστί, the standard term in accounts of human dismemberment. An intriguing idea: could Euphorion be suggesting that the murderous Titans cut the god into a 'perfect' number of pieces?

He is being unorthodox, if so. The number of pieces into which Dionysus was cut was a topic of interest in the Dionysian mysteries. It was the number seven.[36] Proclus in his *Timaeus* commentary explains how the heart of Dionysus was the one part of him which was undivided, because the heart is the seat of indivisible νοῦς. The rest was divided by the Titans into seven pieces, and Proclus quotes a line from ὁ θεολόγος (the Orphic rhapsodies) on the dismemberment of Dionysus by the Titans: ἑπτὰ δὲ πάντα μέλη κούρου διεμοιρήσαντο.[37] Firmicus Maternus also has a reference to the seven spits on which the god was roasted.[38]

If Euphorion's god was cut into a perfect number, can we reconcile the seven of the rhapsodies with a Euclidean perfect number? Could Dionysus have been cut into 6 + 1 (the heart) pieces rather than the 7 + 1 of the Orphic rhapsodies?[39] Another macabre possibility suggested to me by Edward Hussey is that he was cut into the Orphic

μέλη σπαράξαντες Ἀπόλλωνι ἀδελφῷ ὄντι αὐτοῦ παρέθεντο ἐμβαλόντες λέβητι, ὁ δὲ παρὰ τῷ τρίποδι ἀπέθετο, ὥς φησι Καλλίμαχος (fr. 643 Pf.) καὶ Εὐφορίων λέγων· ἐν πυρὶ Βάκχον δῖον ὑπὲρ φιάλης ἐβάλοντο. See M. L. West, *The Orphic Poems* (Oxford, 1983), pp. 150–2, 160–3. According to West, it was the Orphic account which included the grave of Dionysus in Delphi and the preservation of the separate heart (cf. also Call. fr. 517/643 Pf.), and the non-Orphic one which had him fitted back together 'on the same body or at least on the same skeleton'. So both Orphic and non-Orphic accounts are known to Euphorion.

33 G. Schultze, *Euphorionea* (Diss. Straßburg, 1888), p. 57; F. Scheidweiler, *Euphorionis Fragmenta* (Diss. Bonn, 1908), p. 27.

34 Plat. *Leg.* 795e τὰ τοῦ σώματος μέλη καὶ μέρη, Arist. *H.A.* 486a 9ff. τῶν δὲ τοιούτων ἔνια οὐ μόνον μέρη ἀλλὰ καὶ μέλη καλεῖται.

35 A. Barigazzi, 'Il Dionysos di Euforione', in *Miscellanea di Studi Alessandrini* (Torino, 1963), pp. 416–54. He discusses this fragment on pp. 447–8: 'Una citazione di Euforione ἐν Μοψοπίᾳ, relativa, mi par cosa sicura, allo squartamento di Bacco, al quale vien riportato secondo una tradizione pitagorica l'origine dei numeri perfetti.'

36 Osiris, in many respects the prototype of the dismembered Dionysus, is said to have been cut up into fourteen pieces, a number given a lunar interpretation; but other traditions allude to 16, 26, and even 42 pieces (see Plut. *Mor.* 358a and 368a, and Gwyn Griffiths' commentary on *De Iside et Osiride*, pp. 338–9).

37 Proclus on Plat. *Tim.* 35a (II 145.18 Diehl, Orph. fr. 210 Kern).

38 Firm. Mat. *De Error.* 8.2: *alii crudeli morte caesum aut in olla decoquunt aut septem veribus corporis mei membra lacerata subfigunt*, cited in A. Henrichs, *Die Phoinikika des Lollianos* (Bonn, 1972), pp. 67–8, n. 59.

39 See W. Burkert, *Homo Necans: The Anthropology of Ancient Greek Sacrificial Ritual and Myth* (Berkeley/Los Angeles, 1983), p. 105 with n. 11 on forms of dismemberment which are relics of hunting rituals.

number of seven pieces, and then that each piece was the cut into a further four, thus producing, not six, but the second perfect number, 28. And after that, the number of exotic forms of dismemberment which can be envisaged is limited only by the imagination of the reader. But why should a division into six pieces result in the designation *τέλειοι*? The implication may be that Euphorion is offering an etymology for perfect numbers. If Barigazzi is right about the context, then one possibility comes to mind. The Dionysiac mysteries concerned with the death and rebirth of Dionysus were called *τελεταί*. Perhaps Euphorion is punning on *τέλειος* and *τελετή* (or even on the use of *τέλειος* in sacrifices to denote unblemished sacrificial victims?). *τελετή* is easy to pun on, and Plato makes a similar connection in Phaedr. 249c: *τελέους ἀεὶ τελετὰς τελούμενος τέλεος ὄντως . . . γίγνεται*. Does Euphorion mean that 'perfect' numbers originated with the 'perfect' dismemberment of Dionysus, henceforth celebrated in a rite called the *τελεταί*?

But this is only one possibility. Another is that the *μέλη* are those of the assailants, not of the victim. The Titans and Giants had six hands each, according to Orph. *Arg.* 517 *ἓξ γὰρ χεῖρες ἑκάστῳ ἀπ' ὤμων ἀίσσοντο* (an adaptation of Hes. *Theog.* 150, on the hundred-handers Cottus, Briaereus, and Gyges). This is unusual: Titans and Giants are normally only endowed with the standard number of hands, and one wonders why the Orphic poet or his source made the change. A third possibility, of course, is that the context in Euphorion is not about dismemberment at all, but (perhaps) concerns medicine—healing, or the sort of subject which might have justified the line's inclusion in a medical compilation. But there is little point in further speculation.

CONCLUSION

The foregoing was conjecture. There is no indication of context in the source who quotes the line, and nothing about Dionysus in the line itself. What is more important is the likelihood that here we have a poet in the middle of the third century B.C. referring to a mathematical notion which in extant literature can only be traced back as far as Euclid. Either Euphorion knows Euclid, in which case we have one of our earliest references, perhaps our very earliest, to the *Elements*; or he has picked up material which has already spread through the mathematical community, which is evidence for the pre-Euclidean existence of the idea of perfect numbers. But which?

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